PHYSICS NYB-10/11 Winter 2007

Lecture 6: Electric potential energy

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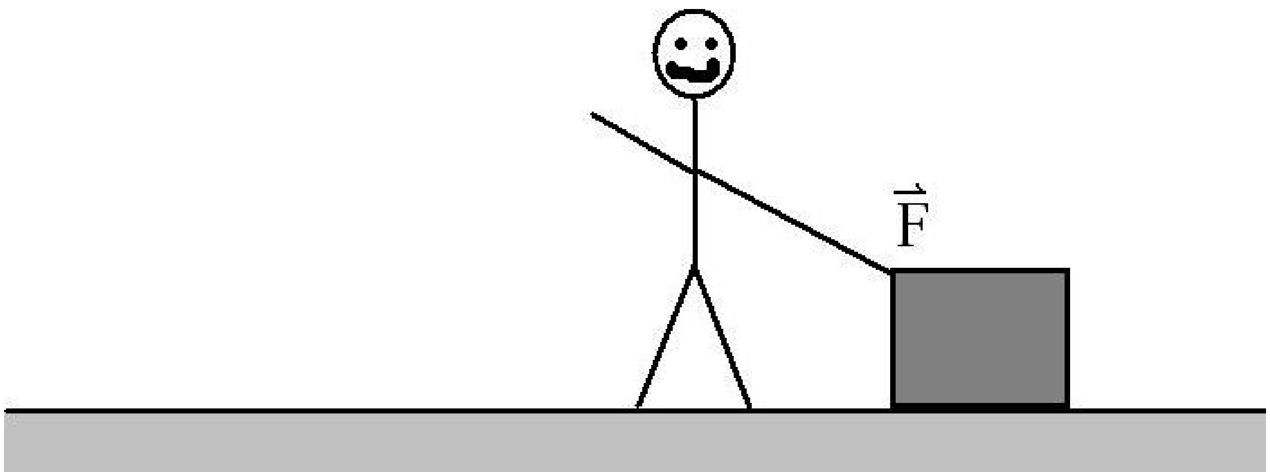
Review

Important points from last lectures:

- A point charge q creates an electric field $\vec{E} = k_e \frac{q}{r^2} \hat{r}$
- A point charge q_0 placed in an electric field $\vec{E}$ feels a force $\vec{F}_e = q_0 \vec{E}$

NYA flashback: Work

- When a net force acts on an object, it accelerates it.
- When an object accelerates, its velocity changes.
- When the speed of an object changes, its kinetic energy changes.
- When an object's energy changes, *work* has been done on it.



NYA flashback: Work

In less cartoonish terms, The amount of work W done by a force $\vec{F}$ over a displacement $\Delta\vec{r}$ is

$$W = \vec{F} \cdot \Delta\vec{r}$$

The amount of work W done by a force $\vec{F}$ over two displacements $\Delta\vec{r}_1$ and $\Delta\vec{r}_2$ is

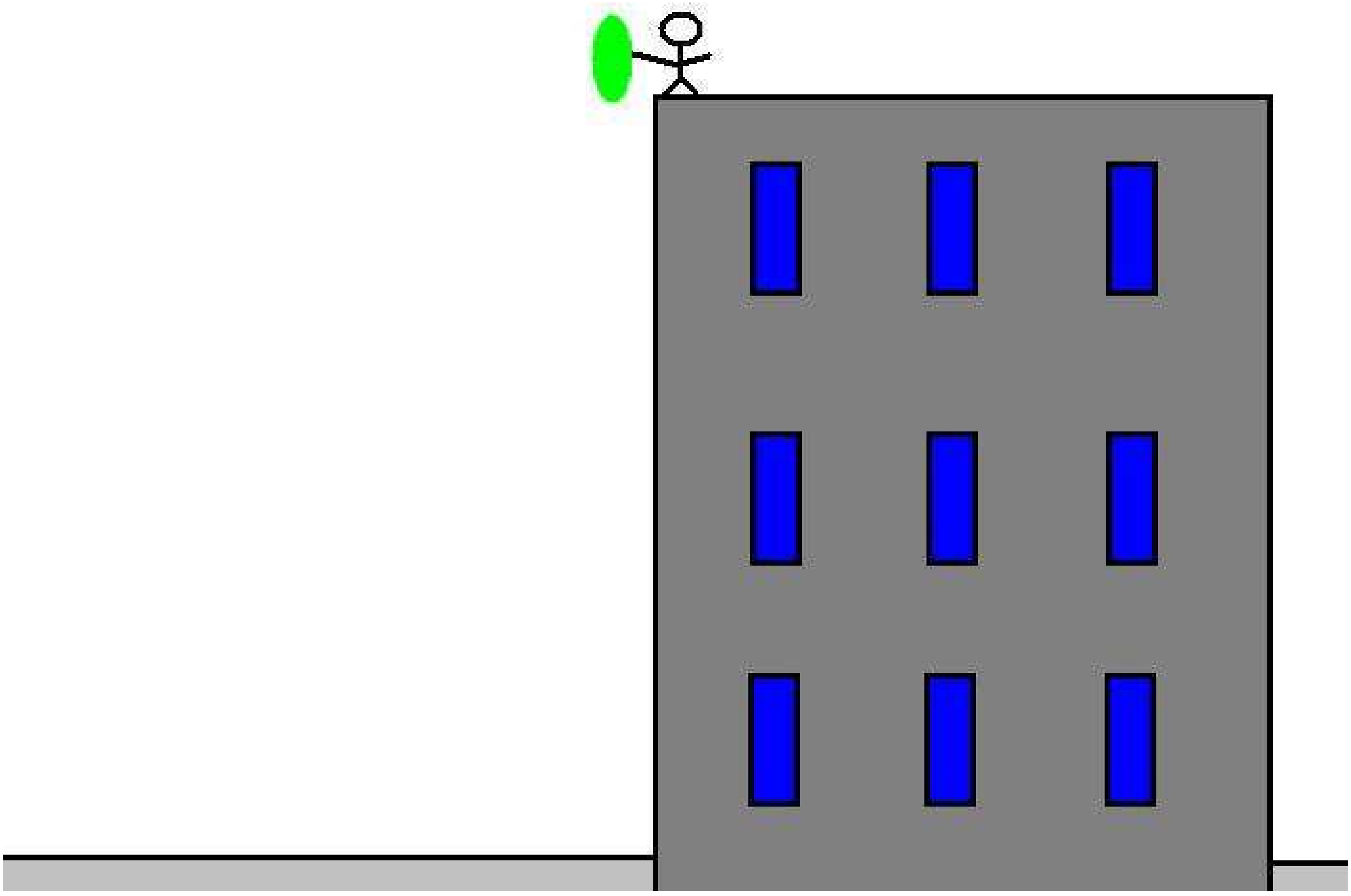
$$W = \vec{F} \cdot (\Delta\vec{r}_1 + \Delta\vec{r}_2)$$

Remember the definition of the dot product

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta = A_x B_x + A_y B_y + A_z B_z}$$

NYA flashback: Work: example

What is the work done by gravity on a watermelon dropped from the College's roof to the parking lot below?



NYA flashback: Work: example

The force acting on the melon as it falls is $\vec{F}_g = -mg\hat{j}$. The displacement it undergoes is $\Delta\vec{r} = -h\hat{j}$. We put these together to find that the work is $W_g = \vec{F}_g \cdot \Delta\vec{r} = (-mg\hat{j}) \cdot (-h\hat{j}) = mgh(\hat{j} \cdot \hat{j}) = mgh$. So the amount of work done by gravity on the melon of mass m as it dropped a distance h is mgh.

(Remember, $\hat{j} \cdot \hat{j} = |\hat{j}||\hat{j}| \cos\theta = 1 \times 1 \times \cos(0) = 1$.)

NYA flashback: Potential energy

- We just saw that if we drop an object of mass m from a height h in a gravitational field of magnitude g, the work done by the field on the object will be $W = mgh$.
- This expression should remind you of something...
- It is the gravitational potential energy of an object of mass m held at a height h above the surface of the Earth.
- *The potential energy is equal to the work a field can do on an object if we release the object*.

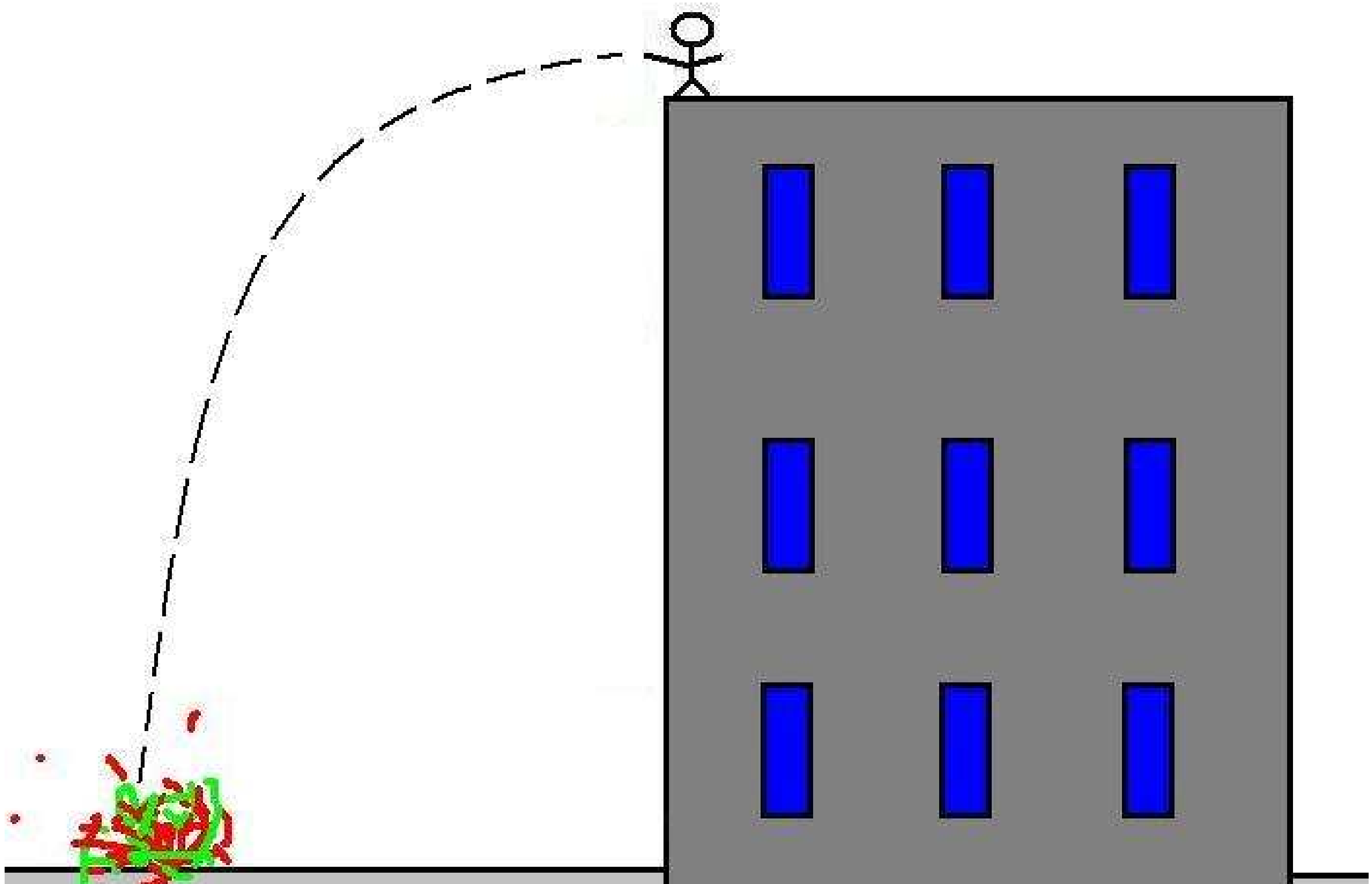
NYA flashback: Potential energy

Things you should remember about potential energy:

- Only the *difference* in the potential energy between two points matters, not the actual values.
- We can therefore set U to zero wherever we please.
- The work done by the force is equal to minus the change in the potential energy from point A to point B ; $W_g = -\Delta U_g$.
- For a *conservative* force (like the gravitational for or the electric force), the path taken from point A to point B makes no difference!

NYA flashback: Potential energy

What is the work done by gravity on a watermelon dropped *on a parabolic path* from the College’s roof to the parking lot below?



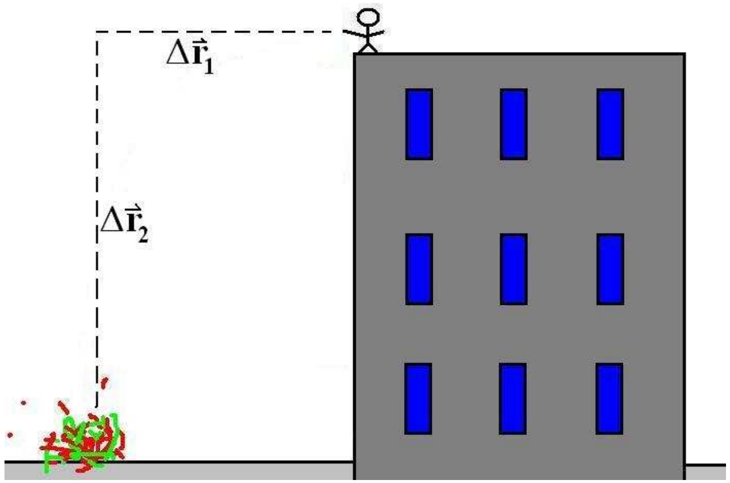
NYA flashback: Potential energy

The only thing that matters here is the initial and final heights, since the potential energy is $U_g = mgh$. So the work done by gravity on the object is again $W_g = mgh$. Note that this is minus the change in the potential energy $\Delta U_g = U_f - U_i = 0 - mgh = -mgh = -W_g$, as it should.

Notice that since we are free to set $U_g = 0$ wherever we want, we could have said that $U_g = 0$ at the college's roof. In this case, however, the potential energy at the ground is $U_g = -mgh$, and we still find $\Delta U_g = U_f - U_i = -mgh - 0 = -mgh = -W_g$, which confirms that the choice of where $U_g = 0$ doesn't change the answer to the problem.

NYA flashback: Potential energy

Note that the work done doesn't depend on the path taken from the initial to the final point, so we could find the work by looking at the path shown here,



where clearly the work done on the horizontal stretch by gravity is zero since the angle between force and displacement is 90^o, and the work done on the vertical stretch is mgh.

Work and the electric field

If we place a charge q_0 in an electric field $\vec{E}$, the electric field can do work on the charge. Indeed, in this case, the force on the charge is $\vec{F} = q_0 \vec{E}$, so that the work done on q_0 by the electric field is

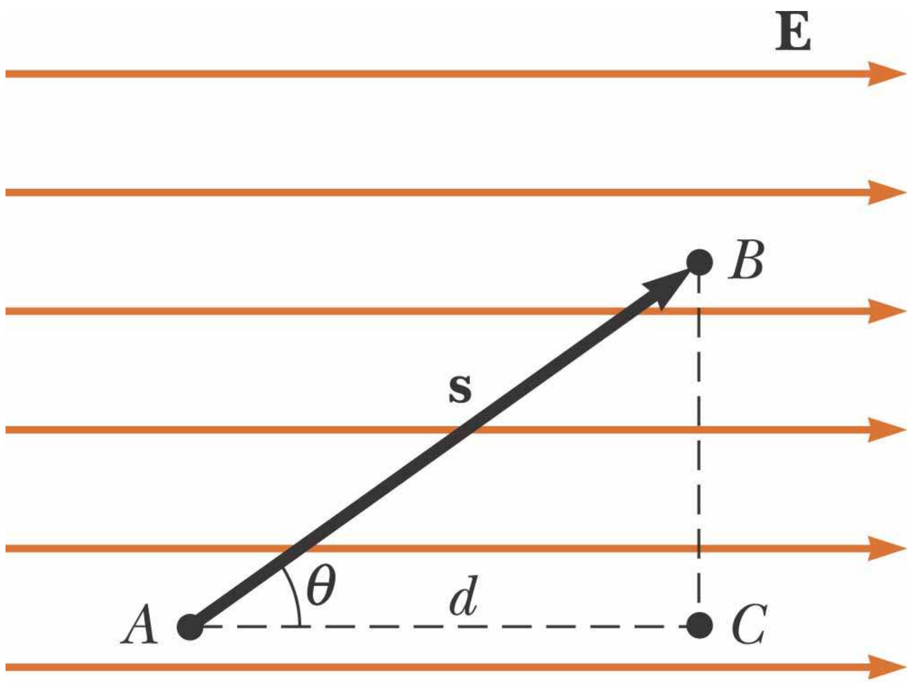
$$W_e = \vec{F}_e \cdot \Delta \vec{r} = q_0 \vec{E} \cdot \Delta \vec{r}$$

and the change in the potential energy of the charge-field system is

$$\Delta U_e = -W_e = -q_0 \vec{E} \cdot \Delta \vec{r}$$

Work and the electric field

A particle of charge $q_0 = 5$ nC moves from point A to point B (where $d = 10$ cm and $\theta = 30^o$) in a uniform electric field $\vec{E} = E\hat{i} = 100\hat{i}$ N/C as shown below. Find the work done by the field on the charge over this displacement. What is the change in the potential energy? Would the answer differ the particle had gone from A to C then from C to B rather than from A to B directly?



Work and the electric field

The displacement vector $\Delta\vec{r} = d\cos\theta\hat{i} + d\sin\theta\hat{j}$. The force exerted by the field on the particle is $\vec{F}_e = q\vec{E} = qE\hat{i}$, so the work done by this force over this displacement is

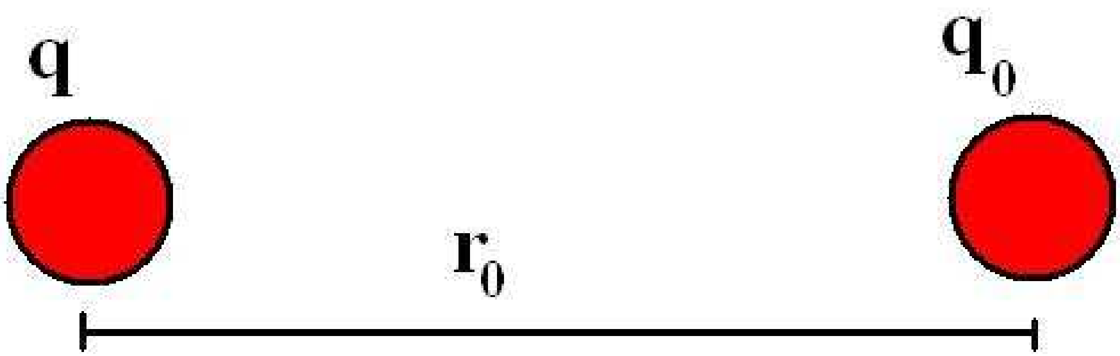
$$\begin{aligned} W_e &= \vec{F}_e \cdot \Delta\vec{r} = qE\hat{i} \cdot (d\cos\theta\hat{i} + d\sin\theta\hat{j}) \\ &= qEd\cos\theta(\hat{i}\cdot\hat{i}) + qEd\sin\theta(\hat{i}\cdot\hat{j}) = qEd\cos\theta \times 1 + 0 \\ &= qEd\cos\theta = 4.33 \times 10^{-8}\ \text{J} \end{aligned}$$

This answer would not change if we had taken a different path from A to B.

Note that since the work done by the electric field is 4.33×10^{-8} J, we know that the potential *electric* energy must have changed by $\Delta U_e = -4.33 \times 10^{-8}$ J.

Electric potential energy

Imagine we have a positive point charge q located at the origin of a coordinate system, and a second positive point charge q_0 located a distance r_0 away. We let q_0 go. Find the work done on q_0 by the electric field associated with q as it accelerates to $r = \infty$.



Work from a non-constant force

Before solving the problem from the previous slide, we have to figure out what to do when the force is not constant. Indeed, the electric field from the charge q is

$$\vec{E} = k_e \frac{q}{r^2}\hat{r},$$

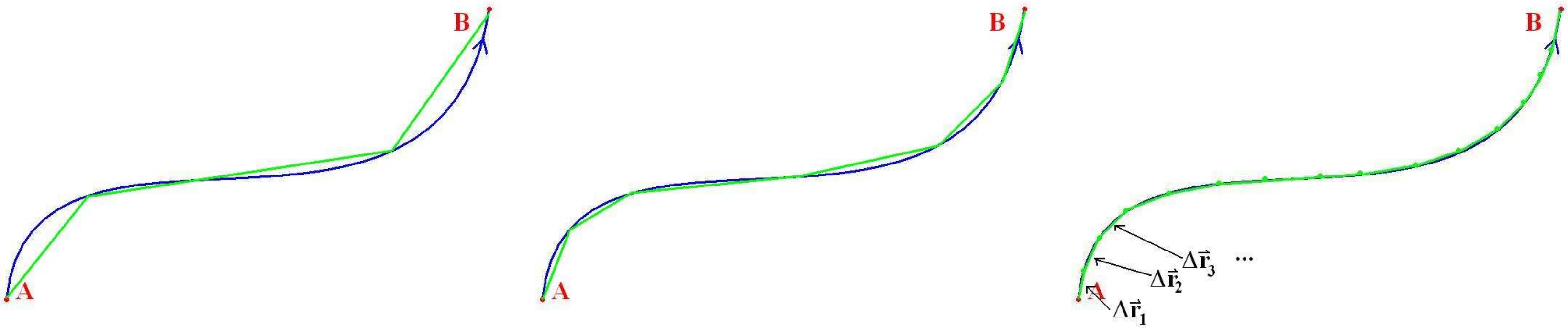
which means the force on the charge q_0 placed in this field is

$$\vec{F}_e = q_0\vec{E} = k_e \frac{q_0 q}{r^2}\hat{r}$$

This force decreases as q_0 gets farther away from q...

Work from a non-constant force

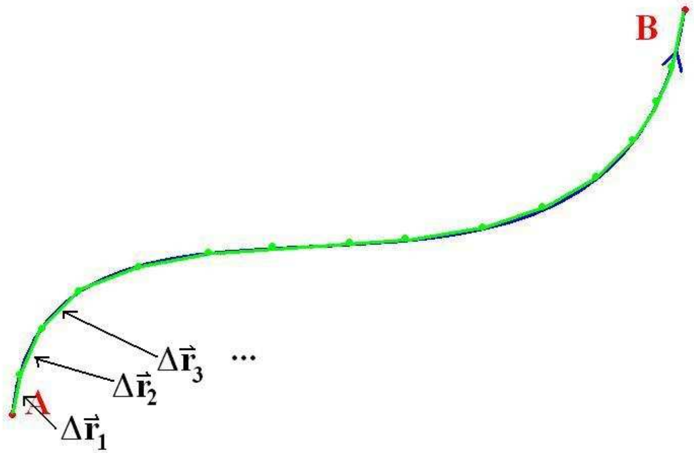
Luckily, we can break down any displacement into a bunch of small displacements



Over each small displacement Δr_i the force is almost constant, and the work done by the force is

$$W_i = \vec{F}_i \cdot \Delta\vec{r}_i$$

Work from a non-constant force



The total work done over the whole path is the sum of the work done on each small path.

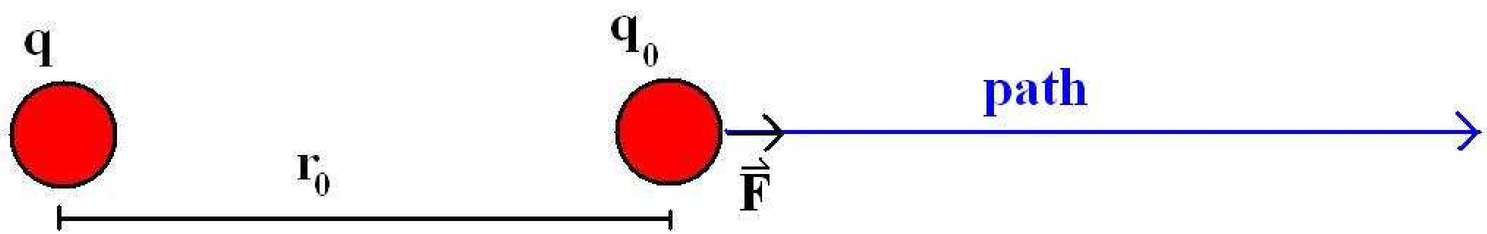
$$W = \sum_i \vec{F}_i \cdot \Delta\vec{r}_i$$

If we make the small displacements infinitely small, we get an infinite sum, or an integral!

$$\boxed{W_e = \int_A^B \vec{F}_e \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}}$$

Electric potential energy

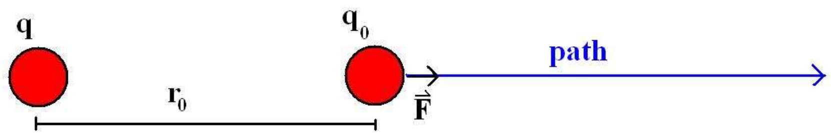
We are now ready to solve the problem:



If the charge q_0 starts from rest, it will accelerate to the right, so that its displacement will everywhere be parallel to the force, which means $\vec{F} \cdot d\vec{s} = F\,dr$.

Electric potential energy

We are now ready to solve the problem:



We can now find the total work done through

$$
\begin{aligned}
W_e &= \int_A^B \vec{F}_e \cdot d\vec{s} = \int_{r_0}^{\infty} k_e \frac{q_0 q}{r^2} dr \\
&= k_e q_0 q \int_{r_0}^{\infty} \frac{dr}{r^2} = k_e q_0 q \left[\frac{-1}{r} \right]_{r_0}^{\infty} \\
&= k_e q_0 q \left[0 - \frac{-1}{r_0} \right] = k_e \frac{q_0 q}{r_0}
\end{aligned}
$$

Note that this is a finite value!

Electric potential energy

So we find that the work done by the electric field of the charge q on the charge q_0 as it goes from a distance r_0 to infinity is

$$W_e = k_e \frac{q_0 q}{r_0}$$

We also know that this amount of work is minus the change in the potential energy, so

$$\Delta U_e = U_f - U_i = -W_e = -k_e \frac{q_0 q}{r_0}$$

We now *choose* to set $U_e = 0$ at infinite separation so that $U_f = 0$, and we find that $U_i = k_e \frac{q_0 q}{r_0}$.

Electric potential energy

- The potential energy between two charges q and q_0 separated by a distance r is

$$\boxed{U_e(r) = k_e \frac{q_0 q}{r}}$$

- $E_{tot} = K + U =$ constant
- U_e is an energy, so it is measured in *joules*, J.
- By convention, we will set $U_e = 0$ when $r = \infty$
- Charges which start from rest and are free to move will move so as to minimize $U(r)$ (and increase K)
- $U_e(r)$ is positive for like charges (which repel), negative for opposites (which attract)

Examples

A small positive charge is moving at speed v directly towards a large positively charged particle. Explain qualitatively what is going to happen from the point of view of forces, fields and energy.

Examples

From the point of view of forces, there is a repulsive force on the moving charge which leads to an acceleration in the negative x-direction, so that its speed will decrease.

From the point of view of fields, the moving charge is immersed inside an electric field, and it is this field which leads to the moving charge feeling a force.

From the point of view of energy, the moving charge has kinetic energy, and potential energy due to the presence of the other charge. As it approaches the other charge, the potential energy becomes larger, so conservation of energy implies that its kinetic energy decreases, so it will slow down.

Examples

A small positively charge is moving at speed v directly towards a large negatively charged particle. Explain qualitatively what is going to happen from the point of view of forces, fields and energy.

Examples

From the point of view of forces, there is an attractive force on the moving charge which leads to an acceleration in the positive x-direction, so that its speed will increase.

From the point of view of fields, the moving charge is immersed inside an electric field, and it is this field which leads to the moving charge feeling a force.

From the point of view of energy, the moving charge has kinetic energy, and negative potential energy due to the presence of the other, opposite charge. As it approaches the other charge, the potential energy becomes more and more negative, so conservation of energy implies that its kinetic energy increases, so it will speed up.

Examples

A 4 mm diameter plastic bead is charged to -1 nC.

a) An alpha particle is fired at the bead from far away with a speed of 1×10^6 m/s and it collides head-on. What is the impact speed?

b) An electron is fired at the bead from far away. It "reflects", with a turning point 0.1 mm from the surface of the bead. What was the electron's initial speed?

Examples

The bead is a sphere, so outside its radius, it behaves like a point charge. The potential energy when the alpha particle is far away is essentially 0. Meanwhile, the alpha particle ($q = +2e$ and $m = 2m_p + 2m_n$) has kinetic energy $K = \frac{1}{2}mv_i^2$. As it approaches the bead, the total energy $K + U = E_{tot}$ is conserved. The potential energy is $U(r) = -k_e\frac{Qq}{r}$. So the kinetic energy is $K = E_{tot} - U = K_i - U$. So the kinetic energy when the alpha particle hits the bead of radius 2 mm is

$$\begin{aligned} \frac{1}{2}mv_f^2 &= \frac{1}{2}mv_i^2 + k_e\frac{Qq}{r} \\ \Rightarrow v_f = &= \sqrt{v_i^2 + 2k_e\frac{Qq}{rm}} = 1.20 \times 10^6 \text{ m/s} \end{aligned}$$

Examples

For part b), the final kinetic energy $K_f = 0$, and we want to find the kinetic energy far away, where $U_i = 0$. We know that $U = k_e \frac{eQ}{r}$, and $K_f + U_f = K_i + U_i$ so that $K_i = U_f$, and

$$
\begin{aligned}
\frac{1}{2} m_e v_i^2 &= k_e \frac{eQ}{r_f} \\
\Rightarrow v_i &= \sqrt{2k_e \frac{eQ}{m_e r_f}} = 3.88 \times 10^7 \text{ m/s}
\end{aligned}
$$

What to read for next lecture

- 25.3, 25.4